Spiral plat avec une seule courbe terminale externe

Anisochronisme en position horizontale

Cas d'une montre bracelet

Caractéristiques du spiral

Référence :E:\Résonateur (TA)\Data\Bal_spiral plat (ex num).mcd(R)

Référence :E:\Résonateur (TA)\Data\Définition Atan.mcd(R)

Dimensions $ép = 0.03\,mm$ $ha = 0.15\,mm$ $S = 4.5\times 10^{-3}\,mm^2$ $TOL := 10^{-12}$

$d2_{sp} = 4.52\,mm$ $d1_{sp} = 1.1\,mm$ $p_{sp} = 0.135\,mm$ $n_{sp} = 12.667$

$L := L_{sp}$ $L = 11.182\,cm$ $\psi_0 := 2\cdot\pi\cdot n_{sp}$ $\psi_0 = 4.56\times 10^3\,deg$

Position du point de raccordement sur le spiral $\alpha_A := \pi$ $r_A := 0.5\cdot d2_{sp}$ $z_A := r_A\cdot e^{i\cdot\alpha_A}$

Forme initiale du spiral

$$a := \frac{p_{sp}}{2\cdot\pi} \qquad r_s(\alpha) := r_A - a\cdot(\alpha-\alpha_A) \qquad x_{0s}(\alpha) := r_s(\alpha)\cdot cos(\alpha) \qquad y_{0s}(\alpha) := r_s(\alpha)\cdot sin(\alpha)$$

$$s(\alpha) := \frac{1}{2\cdot a}\cdot\left(r_A^{\,2} - r_s(\alpha)^2\right) \qquad s(\alpha) := r_A\cdot(\alpha-\alpha_A) - \frac{a}{2}\cdot(\alpha-\alpha_A)^2$$

Courbe terminale externe

$$r_{t1} := 0.8 \qquad r_{t1} := racine\left[(2\cdot r_{t1}-1)^4 - 4\cdot(1-r_{t1})^4 - \pi^2\cdot r_{t1}^{\,2}\cdot(1-r_{t1})^2, r_{t1}\right]\cdot r_A \qquad r_{t1} = 0.832\,r_A$$

$$r_{t2} := 2\cdot r_{t1} - r_A \qquad r_{t2} = 0.665\,r_A \qquad \beta_0 := arctan\left[\frac{\pi\cdot r_{t1}}{2\cdot(r_A - r_{t1})}\right] \qquad \beta_0 = 82.695\,deg \qquad l_t := r_{t2}\cdot\beta_0 + \pi\cdot r_{t1}$$

$$x_{0t1}(\alpha_t) := -r_A + r_{t1}\cdot(1+cos(\alpha_t)) \qquad y_{0t1}(\alpha_t) := r_{t1}\cdot sin(\alpha_t)$$

$$x_{0t2}(\beta_t) := r_{t2}\cdot cos(\beta_t) \qquad y_{0t2}(\beta_t) := r_{t2}\cdot sin(\beta_t)$$

Position des goupilles de raquettes $r_{GR} := r_{t2}$ $\alpha_{GR} := -\beta_0$ $\alpha_{GR} = -82.695\,deg$

$x_{GR} := x_{0t2}(\alpha_{GR})$ $y_{GR} := y_{0t2}(\alpha_{GR})$

Position du point d'attache à la virole $r_V := 0.5\cdot d1_{sp}$ $\alpha_V(\theta) := \alpha_A + \psi_0 + \theta$ $r_B := r_V$

$x_V(\theta) := r_V\cdot cos(\alpha_V(\theta))$ $y_V(\theta) := r_V\cdot sin(\alpha_V(\theta))$ $L_t := L + l_t$

Amplitude stationnaire du balancier $\theta_0 = 270\,deg$

Moment quadratique de section

Référence :E:\Résonateur (TA)\Tables\Modules J, I et W des barres élastiques.mcd(R)

$I_{33} := I_{f\_rect}(ép, ha)$

Déplacement de la virole libre

Contribution du spiral sans ses courbes terminales

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$$s_s(\alpha) := s(\alpha) + l_t \qquad z_{0s}(\alpha) := x_{0s}(\alpha) + i\cdot y_{0s}(\alpha) \qquad f_s(\theta,\alpha) := i\cdot\theta\cdot exp\left(i\cdot\theta\cdot\frac{s_s(\alpha)}{L_t}\right)$$

$$\Delta_{\mathbf{S}}(\theta) := \frac{1}{L_t}\cdot\int_{\pi}^{\pi+\psi_0} z_{0s}(\alpha)\cdot f_s(\theta,\alpha)\cdot r_s(\alpha)\,d\alpha \qquad \Delta_{\mathbf{S}}(\theta_0) = 0.186 + 0.048i\,mm$$

Approximation $\quad \mathbf{OA} := r_A \cdot e^{i\cdot\pi} \quad \mathbf{OB} := r_B \cdot e^{i\cdot(\pi+\psi_0)} \quad f'_s(\theta,\alpha) := \frac{-\theta^2}{L_t}\cdot r_s(\alpha)\cdot exp\left(i\cdot\theta\cdot\frac{s_s(\alpha)}{L_t}\right)$

$$\Delta_{as}(\theta) := \frac{1}{L_t}\cdot\left[\left[\left(i\cdot r_A + 2\cdot a\right)\cdot f_s(\theta,\pi) - r_A\cdot f'_s(\theta,\pi)\right]\cdot\mathbf{OA} + \left[-\left(i\cdot r_B + 2\cdot a\right)\cdot f_s(\theta,\pi+\psi_0) + r_B\cdot f'_s(\theta,\pi+\psi_0)\right]\cdot\mathbf{OB}\right]$$

$$\Delta_{as}(\theta) := \frac{\theta}{L_t}\cdot e^{i\cdot\theta\cdot\frac{l_t}{L_t}}\cdot\left[\left(-r_A + i\cdot 2\cdot a\right)\cdot\mathbf{OA} - \left(-r_B + i\cdot 2\cdot a\right)\cdot e^{i\cdot\theta\cdot\frac{L}{L_t}}\cdot\mathbf{OB}\right] + \frac{\theta^2}{L_t^2}\cdot e^{i\cdot\theta\cdot\frac{l_t}{L_t}}\cdot\left(r_A^2\cdot\mathbf{OA} - r_B^2\cdot e^{i\cdot\theta\cdot\frac{L}{L_t}}\cdot\mathbf{OB}\right)$$

$\Delta_{as}(\theta_0) = 0.185 + 0.047i\, mm$

Contribution de la courbe terminale externe

$z_{0t1}(\alpha_t) := x_{0t1}(\alpha_t) + i\cdot y_{0t1}(\alpha_t) \qquad z_{0t2}(\beta_t) := x_{0t2}(\beta_t) + i\cdot y_{0t2}(\beta_t)$

$$\Delta_{t2}(\theta) := \frac{i\cdot\theta}{L_t}\cdot r_{t2}\cdot\int_{-\beta_0}^{0} z_{0t2}(\beta_t)\cdot exp\left[i\cdot\frac{\theta}{L_t}\cdot\left[r_{t2}\cdot\left(\beta_0+\beta_t\right)\right]\right]d\beta_t \qquad \Delta_{t2}(\theta_0) = 0.073 + 0.09i\, mm$$

$$\Delta_{t1}(\theta) := \frac{i\cdot\theta}{L_t}\cdot r_{t1}\cdot\int_{0}^{\pi} z_{0t1}(\alpha_t)\cdot exp\left[i\cdot\frac{\theta}{L_t}\cdot\left(r_{t2}\cdot\beta_0 + r_{t1}\cdot\alpha_t\right)\right]d\alpha_t \qquad \Delta_{t1}(\theta_0) = -0.235 - 0.137i\, mm$$

$$\Delta_t(\theta) := \Delta_{t1}(\theta) + \Delta_{t2}(\theta) \qquad \Delta_t(\theta_0) = -0.162 - 0.047i\, mm$$

Approximations

$s_{t2}(\beta_t) := r_{t2}\cdot(\beta_0+\beta_t) \qquad f_{t2}(\theta,\beta_t) := i\cdot\theta\cdot exp\left(i\cdot\theta\cdot\frac{s_{t2}(\beta_t)}{L_t}\right) \qquad f'_{t2}(\theta,\beta_t) := \frac{-\theta^2}{L_t}\cdot r_{t2}\cdot exp\left(i\cdot\theta\cdot\frac{s_{t2}(\beta_t)}{L_t}\right)$

$l_{t2} := r_{t2}\cdot\beta_0 \qquad \mathbf{Og_{12}} := \frac{r_{t2}}{l_{t2}}\cdot\int_{-\beta_0}^{0} z_{0t2}(\beta_t)\,d\beta_t \qquad \mathbf{Og_{22}} := \frac{2\cdot r_{t2}}{l_{t2}^2}\cdot\int_{-\beta_0}^{0} r_{t2}\cdot\beta_t\cdot z_{0t2}(\beta_t)\,d\beta_t$

$$\Delta_{at2}(\theta) := \frac{1}{L_t}\cdot\left(l_{t2}\cdot f_{t2}(\theta,0)\cdot\mathbf{Og_{12}} + f'_{t2}(\theta,0)\cdot\frac{l_{t2}^2}{2\cdot r_{t2}}\cdot\mathbf{Og_{22}}\right) \qquad \Delta_{at2}(\theta_0) = 0.073 + 0.09i\, mm$$

$s_{t1}(\alpha_t) := \left(r_{t2}\cdot\beta_0 + r_{t1}\cdot\alpha_t\right) \qquad f_{t1}(\theta,\alpha_t) := i\cdot\theta\cdot exp\left(i\cdot\theta\cdot\frac{s_{t1}(\alpha_t)}{L_t}\right) \qquad f'_{t1}(\theta,\alpha_t) := \frac{-\theta^2}{L_t}\cdot r_{t1}\cdot exp\left(i\cdot\theta\cdot\frac{s_{t1}(\alpha_t)}{L_t}\right)$

$l_{t1} := r_{t1}\cdot\pi \qquad \mathbf{Og_{11}} := \frac{r_{t1}}{l_{t1}}\cdot\int_{0}^{\pi} z_{0t1}(\alpha_t)\,d\alpha_t \qquad \mathbf{Og_{21}} := \frac{2\cdot r_{t1}}{l_{t1}^2}\cdot\int_{0}^{\pi} r_{t1}\cdot\alpha_t\cdot z_{0t1}(\alpha_t)\,d\alpha_t$

$$\Delta_{at1}(\theta) := \frac{1}{L_t}\cdot\left(l_{t1}\cdot f_{t1}(\theta,0)\cdot\mathbf{Og_{11}} + f'_{t1}(\theta,0)\cdot\frac{l_{t1}^2}{2\cdot r_{t1}}\cdot\mathbf{Og_{21}}\right) \qquad \Delta_{at1}(\theta_0) = -0.236 - 0.141i\, mm$$

$\mathbf{Og_1} := \frac{1}{l_t}\cdot\left(l_{t1}\cdot\mathbf{Og_{11}} + l_{t2}\cdot\mathbf{Og_{12}}\right) \qquad \mathbf{Og_1} = 0.632i\, mm$

$$\Delta_{at}(\theta) := \Delta_{at1}(\theta) + \Delta_{at2}(\theta) \qquad \Delta_{at}(\theta_0) = -0.163 - 0.051i\, mm$$

Contribution du spiral entier

$$\Delta_1(\theta) := \Delta_t(\theta) + \Delta_s(\theta) \qquad \Delta_1(\theta_0) = 0.024 + 7.91i \times 10^{-4}\,mm$$

$$u_1(\theta) := Re(\Delta_1(\theta)) \quad v_1(\theta) := Im(\Delta_1(\theta)) \qquad u_1(\theta_0) = 0.024\,mm \quad v_1(\theta_0) = 7.91 \times 10^{-4}\,mm$$

Approximation

$$\Delta_a(\theta) := \Delta_{at}(\theta) + \Delta_{as}(\theta) \qquad \Delta_a(\theta_0) = 0.021 - 3.769i \times 10^{-3}\,mm$$

Calcul des réactions

$$p2_{0s} := \frac{1}{L_t}\cdot\left(\int_{\pi}^{\pi+\psi_0} x_{0s}(\alpha)^2\cdot r_s(\alpha)\,d\alpha + \int_0^{\pi} x_{0t1}(\alpha_t)^2\cdot r_{t1}\,d\alpha_t + \int_{-\beta_0}^{0} x_{0t2}(\beta_t)^2\cdot r_{t2}\,d\beta_t\right) \qquad p2_{0s} = 1.379\,mm^2$$

$$q2_{0s} := \frac{1}{L_t}\cdot\left[\int_{\pi}^{\pi+\psi_0} y_{0s}(\alpha)^2\cdot r_s(\alpha)\,d\alpha + \int_0^{\pi} y_{0t1}(\alpha_t)^2\cdot r_{t1}\,d\alpha_t + \left(\int_{-\beta_0}^{0} y_{0t2}(\beta_t)^2\cdot r_{t2}\,d\beta_t\right)\right] \qquad q2_{0s} = 1.367\,mm^2$$

$$k_{0s} := \frac{1}{L_t}\cdot\left(\int_{\pi}^{\pi+\psi_0} x_{0s}(\alpha)\cdot y_{0s}(\alpha)\cdot r_s(\alpha)\,d\alpha + \int_0^{\pi} x_{0t1}(\alpha_t)\cdot y_{0t1}(\alpha_t)\cdot r_{t1}\,d\alpha_t + \int_{-\beta_0}^{0} x_{0t2}(\beta_t)\cdot y_{0t2}(\beta_t)\cdot r_{t2}\,d\beta_t\right)$$

$$k_{0s} = -0.012\,mm^2$$

$$\mathbf{S_0} := \frac{L}{E\cdot I_{33}}\cdot\begin{pmatrix} q2_{0s} & -k_{0s} \\ -k_{0s} & p2_{0s} \end{pmatrix} \qquad \mathbf{R'}(\theta) := \mathbf{S_0}^{-1}\cdot\begin{pmatrix} u_1(\theta) \\ v_1(\theta) \end{pmatrix} \qquad \mathbf{R'}(\theta_0) = \begin{pmatrix} 1.109 \times 10^{-5} \\ 2.655 \times 10^{-7} \end{pmatrix} N$$

$$|\mathbf{R'}(\theta_0)| = 1.109 \times 10^{-5}\,N$$

Approximations

$$\sigma 2 := \frac{1}{L_t}\cdot\left[\int_{\pi}^{\pi+\psi_0} (|z_{0s}(\alpha)|)^2\cdot r_s(\alpha)\,d\alpha + \int_0^{\pi} (|z_{0t1}(\alpha_t)|)^2\cdot r_{t1}\,d\alpha_t + \int_{-\beta_0}^{0} (|z_{0t2}(\beta_t)|)^2\cdot r_{t2}\,d\beta_t\right]$$

$$\sigma 2 = 2.745\,mm^2$$

$$R'(\theta) := \frac{E\cdot I_{33}}{L}\cdot\frac{2}{\sigma 2}\cdot\begin{pmatrix} u_1(\theta) \\ v_1(\theta) \end{pmatrix} \qquad R'(\theta_0) = \begin{pmatrix} 1.104 \times 10^{-5} \\ 3.64 \times 10^{-7} \end{pmatrix} N \qquad |R'(\theta_0)| = 1.105 \times 10^{-5}\,N$$

Perturbation de période - spiral non déformé en position de repos

$$X(\theta) := \frac{(|\Delta_1(\theta)|)^2}{\sigma 2} \qquad \gamma(\theta) := \frac{d}{d\theta}X(\theta) \qquad Delta(\theta_0) := \frac{-1}{2\cdot\pi\cdot\theta_0}\cdot\int_0^{2\cdot\pi} \gamma(\theta_0\cdot\cos(\varphi))\cdot\cos(\varphi)\,d\varphi$$

$$\mu(\theta_0) := -86400\cdot Delta(\theta_0) \qquad \boxed{\mu(\theta_0) = 1.384} \qquad \boxed{\mu(180\cdot deg) = 0.903}$$

$$X(\theta) := \frac{(|\Delta_a(\theta)|)^2}{\sigma 2} \qquad \gamma(\theta) := \frac{d}{d\theta}X(\theta) \qquad \delta_a(\theta_0) := \frac{-1}{2\cdot\pi\cdot\theta_0}\cdot\int_0^{2\cdot\pi} \gamma(\theta_0\cdot\cos(\varphi))\cdot\cos(\varphi)\,d\varphi$$

$$\mu_a(\theta_0) := -86400\cdot\delta_a(\theta_0) \qquad \boxed{\mu_a(\theta_0) = 1.466} \qquad \boxed{\mu_a(180\cdot deg) = 1.045}$$

Correction du moment d'inertie du balancier

$C := E \cdot \frac{I_{33}}{L_t}$ $\Delta J_b := \frac{C}{\omega_0^{\,2}} - J_b$ $\Delta J_b = -0.674\,mg \cdot cm^2$ $\frac{\Delta J_b}{J_b} = -6.738\,\%$

Référence :E:\Résonateur (TA)\Le spiral plat\SP avec CT_externe - Approximations de Haag.mcd(R)

$\theta_m := 100 \cdot deg, 120 \cdot deg .. 360 \cdot deg$